Using Student Work to Guide Instruction

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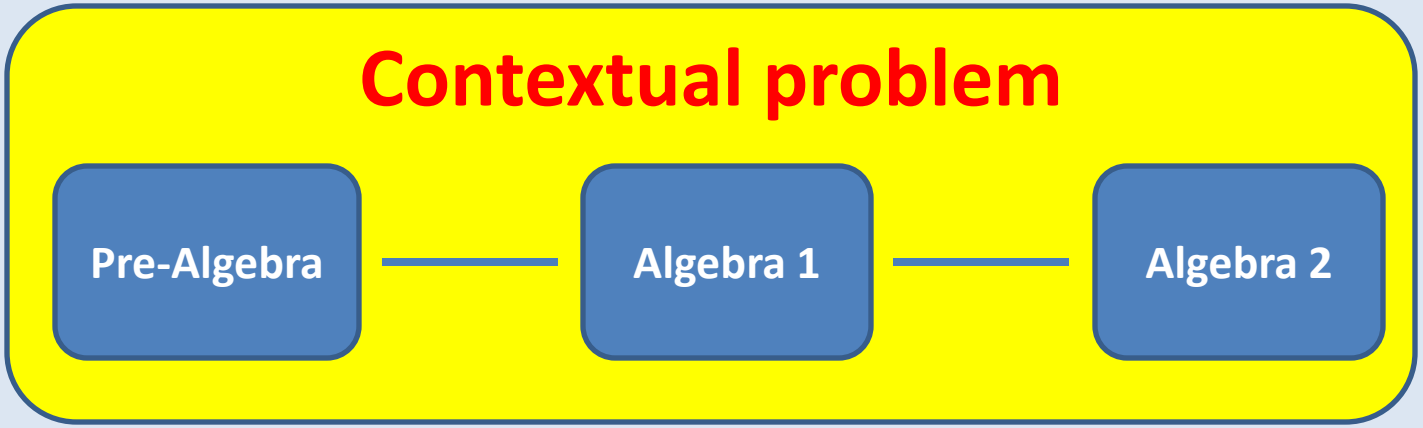
Goals

- Offer activities with multiple access points for students
- Learn how to use student representations
- Interpret student reasoning
- Determine the type of instruction that will be best suited for a particular type of learner

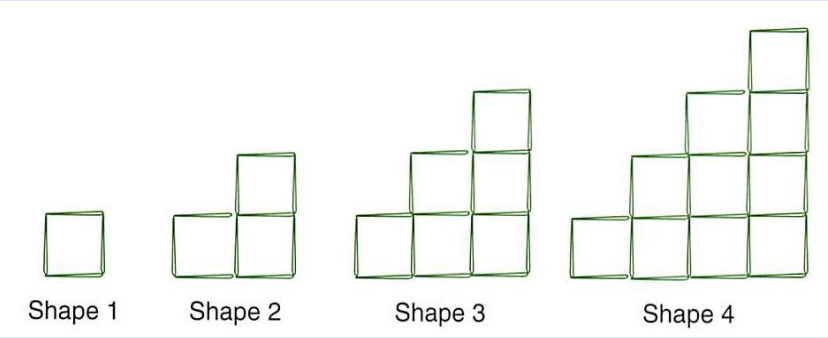
Problems with Access Points

We will look at some problems that have "multiple access points."
Teachers and students can enter at various points of the problem based on their background and readiness.



Pre-Algebra Access Point

Building with Toothpicks



1. Use a pattern from the shapes to determine the perimeter of the fifth shape in the sequence. Show or explain how you arrived at your answer.
2. Write a formula that you could use to find the perimeter of any shape *n*. Explain how you found your formula.

Friel, Susan, Sid Rachlin, and Dot Doyle. Navigating through Algebra in grades 6-8. National Council of Teachers of Mathematics (NCTM): Reston, VA, 2001.

Pre-Algebra Access Point

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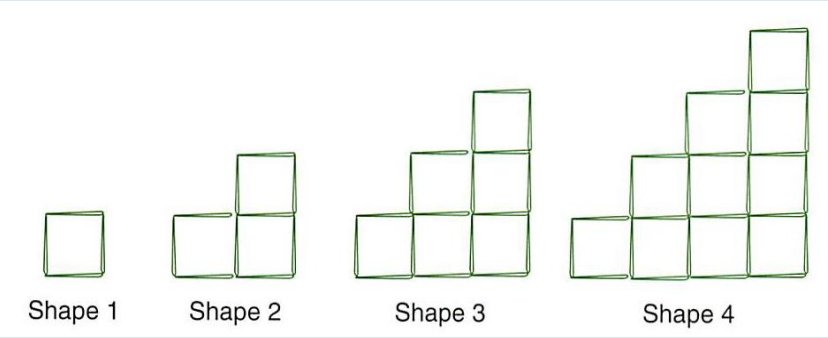
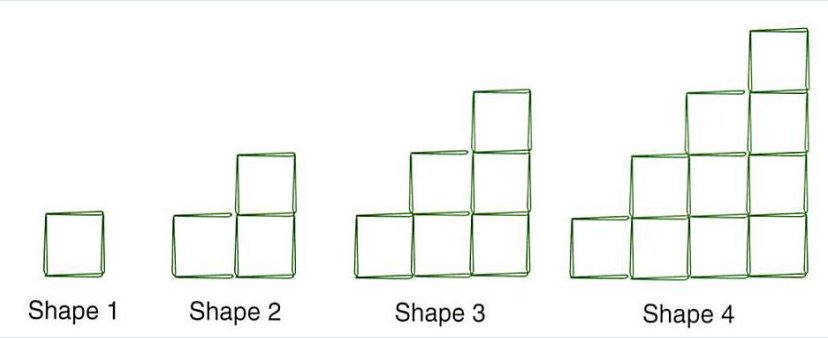


- What type of answers do you expect from students?
- What will you do based on that answer?

Algebra 1 Access Point

Building with Toothpicks



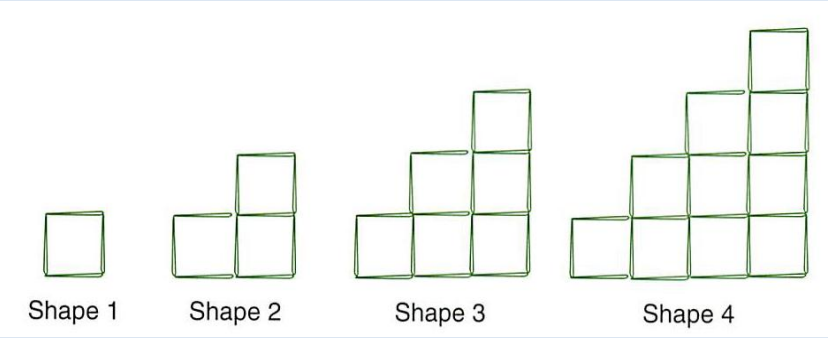
3. Create a table and a graph of the first seven shapes in the pattern. What rule did you use to continue the pattern? Explain how you determined your rule.

4. How would the pattern differ if you used triangles instead of squares?

Extension from: Friel, Susan, Sid Rachlin, and Dot Doyle. Navigating through Algebra in grades 6-8. National Council of Teachers of Mathematics (NCTM): Reston, VA, 2001.

Algebra 1 Access Point

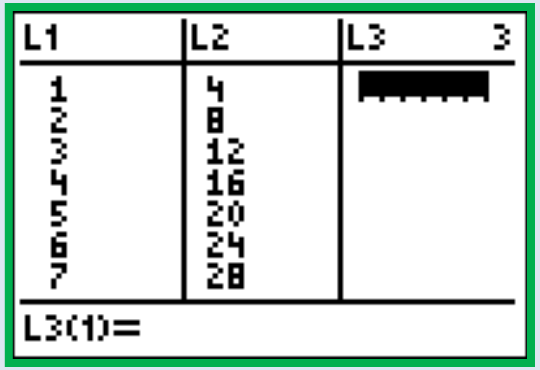
Building with Toothpicks

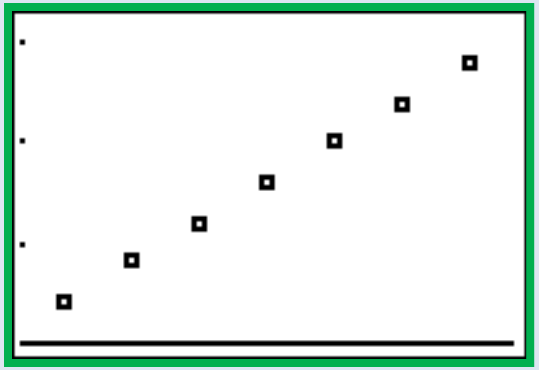


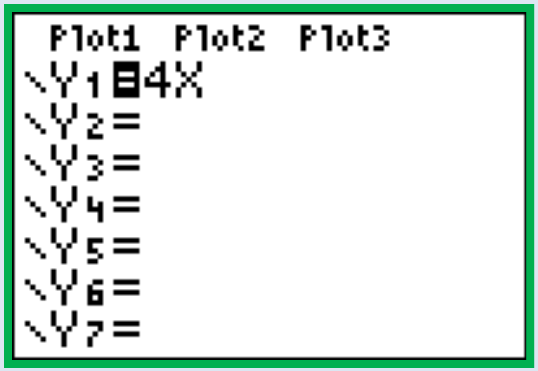
- What type of answers do you expect from students?
- What will you do based on that answer?

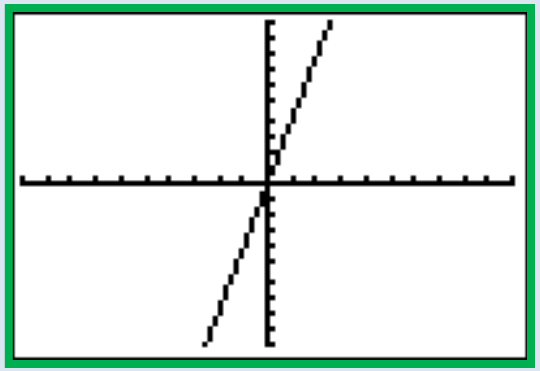
Algebra 1 Access Point

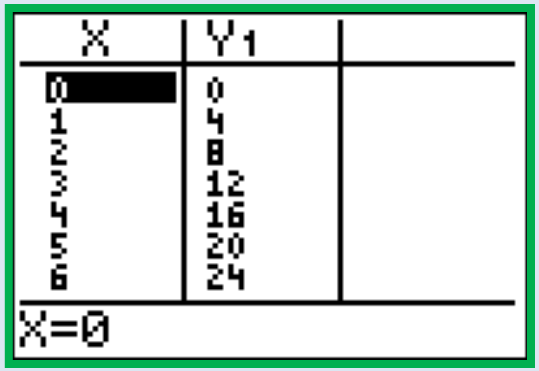
Building with Toothpicks





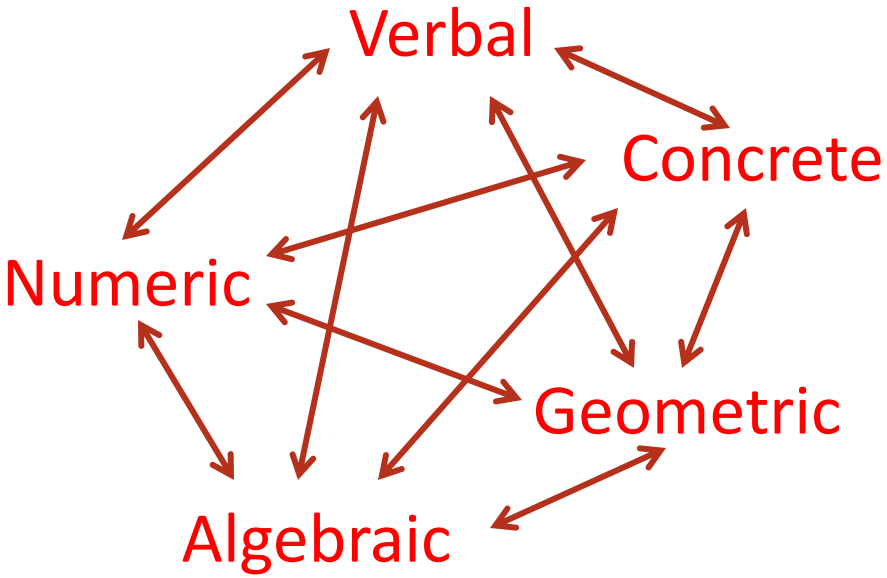






Generalizing

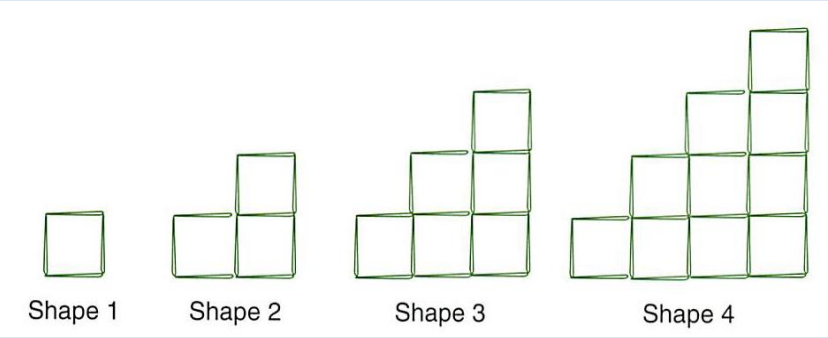
We need to assist our students in moving among representations:



to generalize strategies and solve problems.

Algebra 2 Access Point

Building with Toothpicks



5. Determine the explicit and recursive formulas for finding the perimeter of the n^{th} figure.

6. What would be the perimeter of the 100^{th} figure?

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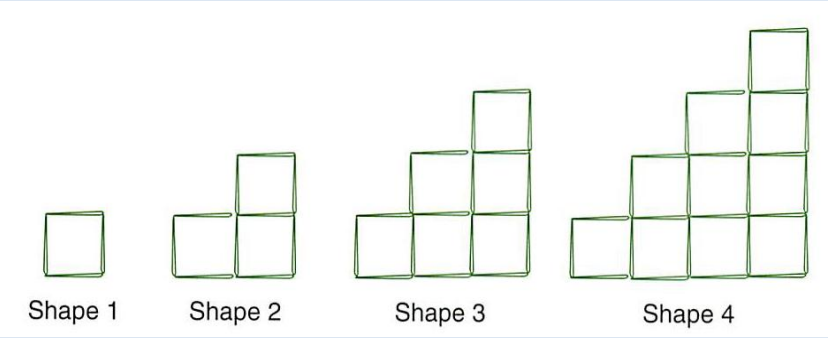
Algebra 2 Access Point

Building with Toothpicks



- What type of answers do you expect from students?
- What will you do based on that answer?

Patterns of Dots

A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues indefinitely.

Step 1

Step 2

Step 3

How do you determine the number of dots in Step 20, but not have to draw all 20 pictures and then count the dots?

Explain or show how you could do this and give the answer that you get for the number of dots.

Problem adapted from Marcy's Dots problem, NAEP 1992.

Solutions?

- What did you do to solve the problem?
- What solution did you get? How can you justify your solution?
- What about the problem allows multiple solutions?

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- What solution did you get? How can you justify your solution?
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Now, let's examine some student work

Examining Student Work

Sample 1

Sample 2

Sample 3

Sample 4

Sample 5

Sample 6

Solution 1

Patterns of Dots

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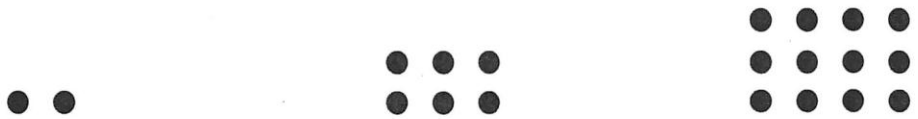
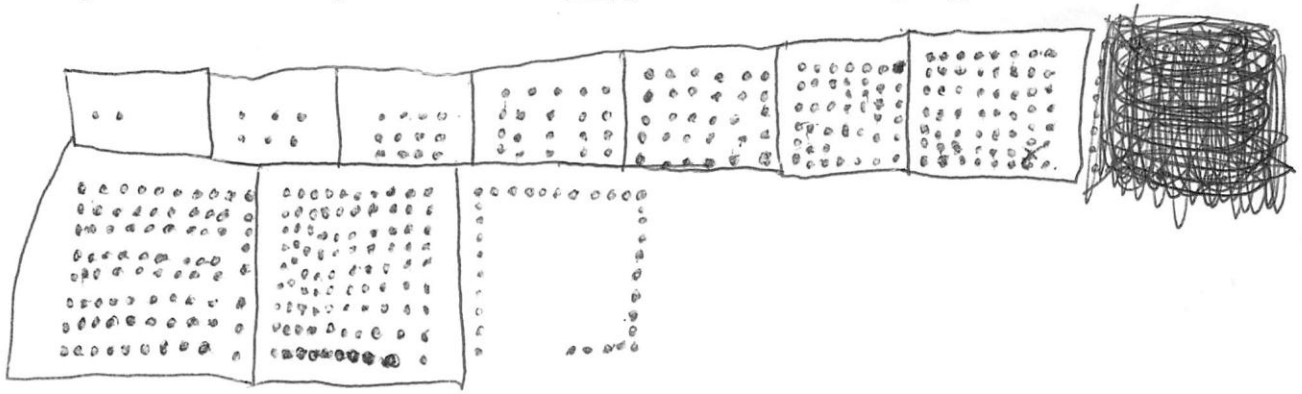
Step 1 Step 2 Step 3



How do you determine the number of dots in Step 20, but not have to draw all 20 pictures and then count the dots?

Explain or show how you could do this and give the answer that you get for the number of dots.



Implications of Solution Type 1

Student started a geometric progression

Next steps could include:

- Asking “What pattern do you notice?”
- Asking “Is there a pattern here? Can you think of a pattern that you could find so that you do not need to draw every step?”
- Move from geometric representation to numeric representation (symbolic).

Sample Menu

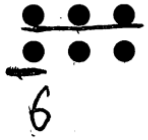
Solution 2

Patterns of Dots

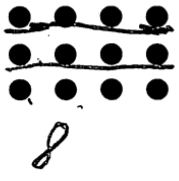
A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues indefinitely.

Step 1

Step 2



Step 3



How do you determine the number of dots in Step 20, but not have to draw all 20 pictures and then count the dots?

Explain or show how you could do this and give the answer that you get for the number of dots.

4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42

I think 42 will be the number of dots.

Implications of Solution Type 2

Student found differences but ignored foundational number

Next steps could include:

- Asking “Why did you record 4, 6, 8?”
- Asking “What is the number of dots in steps 1, 2, and 3?”
- Asking if the answers are reasonable.
- Connecting numeric representation to geometric representation.

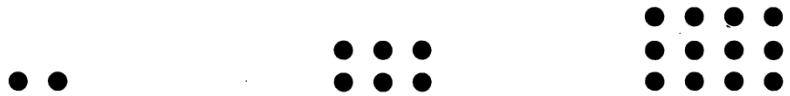
Sample Menu

Solution 3

Patterns of Dots

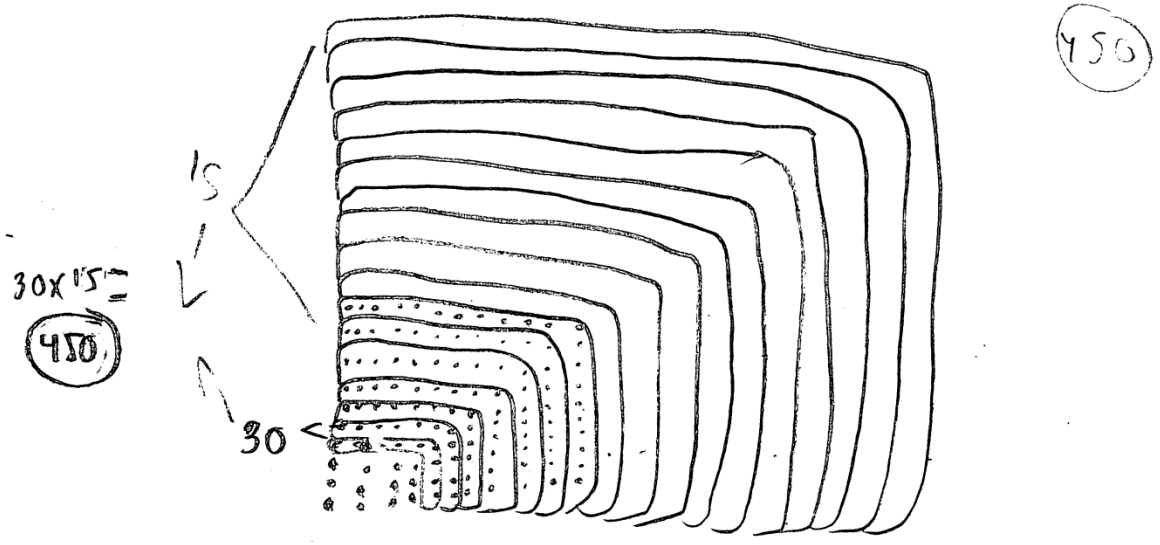
A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues indefinitely.

Step 1 Step 2 Step 3



How do you determine the number of dots in Step 20, but not have to draw all 20 pictures and then count the dots?

Explain or show how you could do this and give the answer that you get for the number of dots.



Implications of Solution Type 3

Student identifies a geometric pattern

Next steps could include:

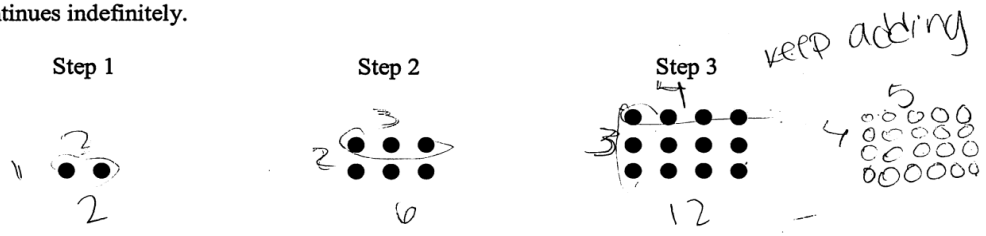
- Asking “How did you get 15 and 30?”
- Asking How does 15 and 30 fit with steps 1, 2, and 3?”
- Move from geometric representation to numeric representation (symbolic).

Sample Menu

Solution 4

A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues indefinitely.

Step 1 | Step 2 | Step 3

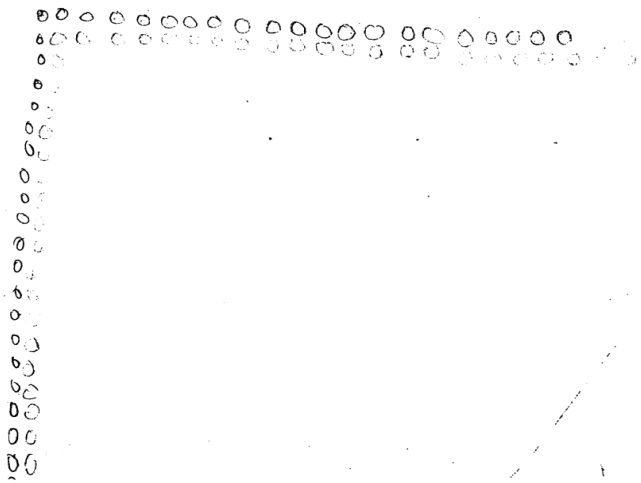


How do you determine the number of dots in Step 20, but not have to draw all 20 pictures and then count the dots?

Explain or show how you could do this and give the answer that you get for the number of dots.

5 | 5,6 | 6,7 | 7,8 | 8,9 | 9,10 | 10,11 | 11,12 | 12,13 | 13,14 | 14,15 | 15,16

16 | 16,17 | 17,18 | 18,19 | 19,20 | 20,21



Implications of Solution Type 4

Student demonstrates geometric and numeric understanding

Next steps could include:

- Asking “What is your actual answer?”
- Asking “Did you check your answer?”
- Move to the next problem

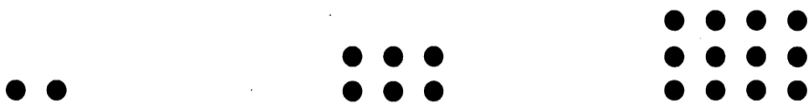
Sample Menu

Solution 5

Patterns of Dots

A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues indefinitely.

Step 1 Step 2 Step 3



How do you determine the number of dots in Step 20, but not have to draw all 20 pictures and then count the dots?

Explain or show how you could do this and give the answer that you get for the number of dots.

2 6 12 16 32 36 72 76 152 156 312 316

+4 ×2 +4 ×2 +4 ×2 +4 ×2 +4 ×2 +4 +2

660

Implications of Solution Type 5

Student found a numeric pattern, which is a repeating pattern instead of a building one, and does not align to the dot progression.

Next steps could include:

- Asking student to show the dot pattern for step 4.
- Asking "Did you determine the 20th step?"
- Connecting numeric representation to geometric representation.

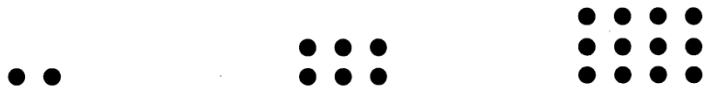
Sample Menu

Solution 6

Patterns of Dots

A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues indefinitely.

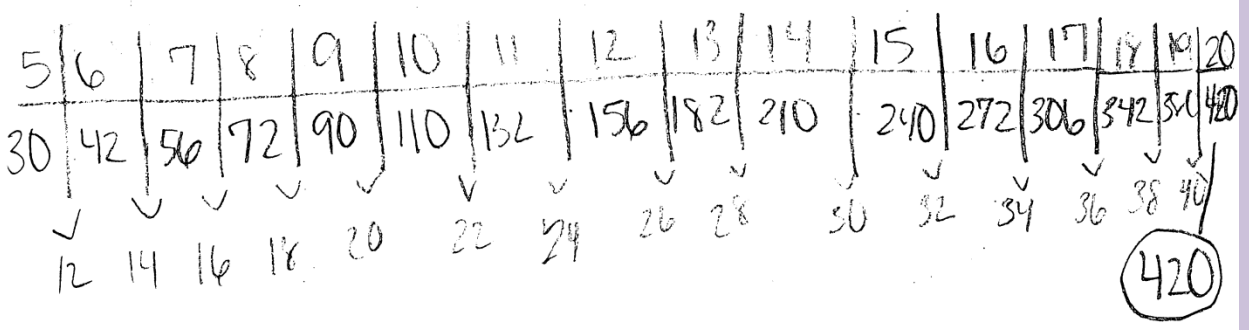
Step 1 Step 2 Step 3



How do you determine the number of dots in Step 20, but not have to draw all 20 pictures and then count the dots?

Explain or show how you could do this and give the answer that you get for the number of dots.

Step 1 has 1 row, step 2 has 2 rows, Step 3 has 3 rows, step 4 has 4 rows and so on. which would mean step 20 would have 20 rows.



Implications of Solution Type 6

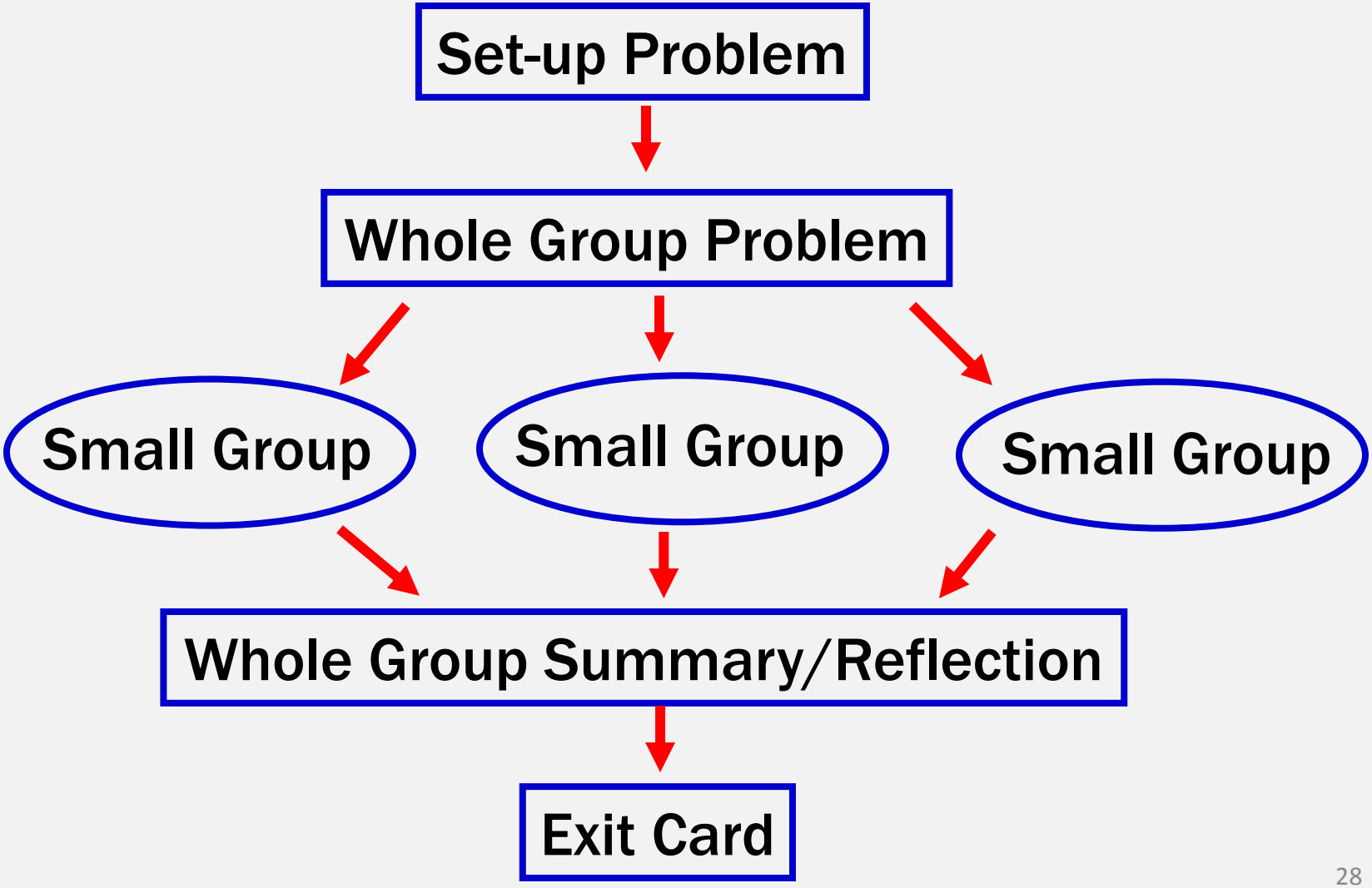
Student demonstrates understanding of differences.

Next steps could include:

- Asking “What would the pattern of dots look like for step 20?”
- Asking “Could you get this answer without determining each step? What about determining the number of dots in step 50?”
- Move to the next problem.

Sample Menu

One Possible Lesson Design



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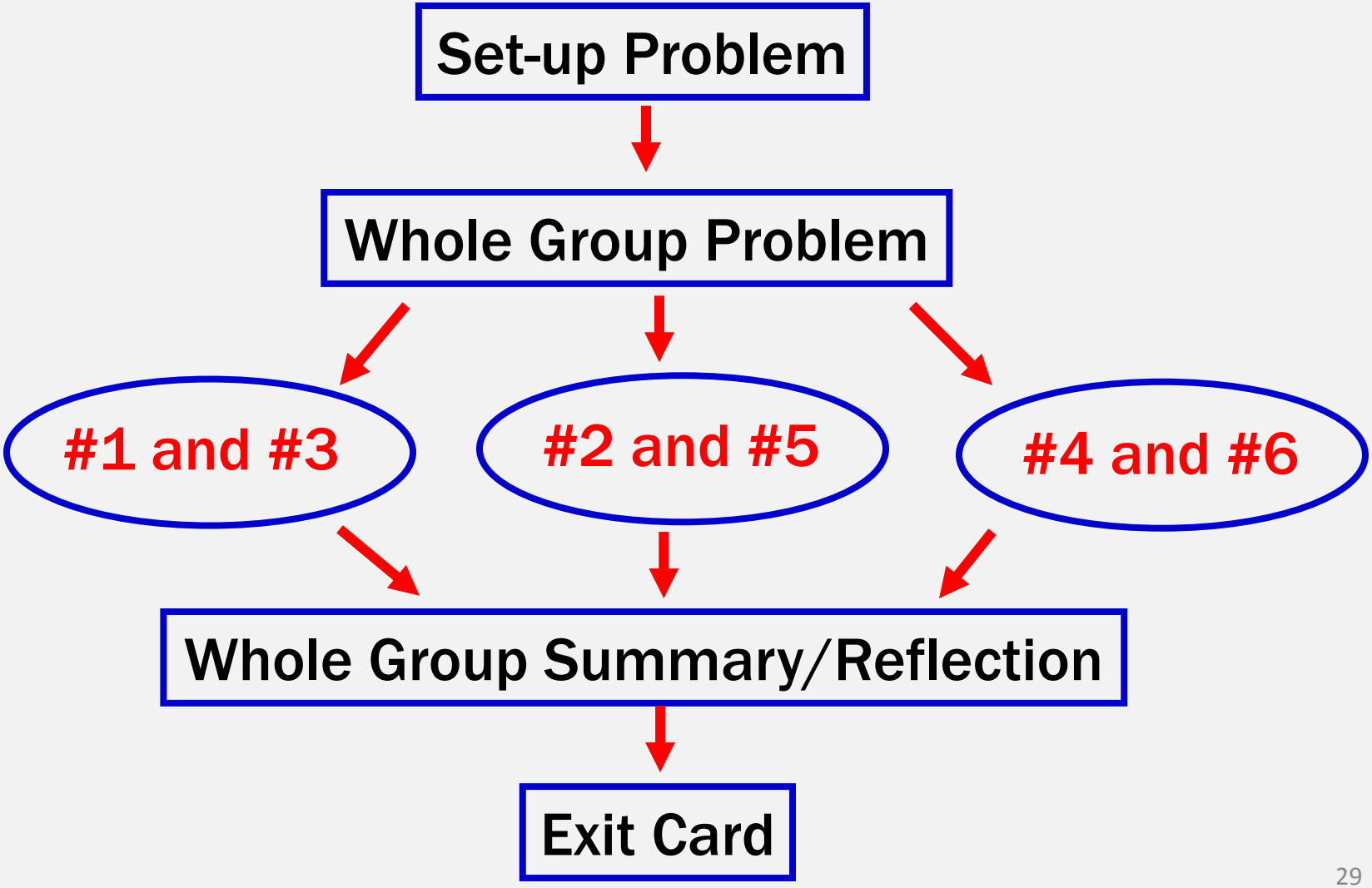
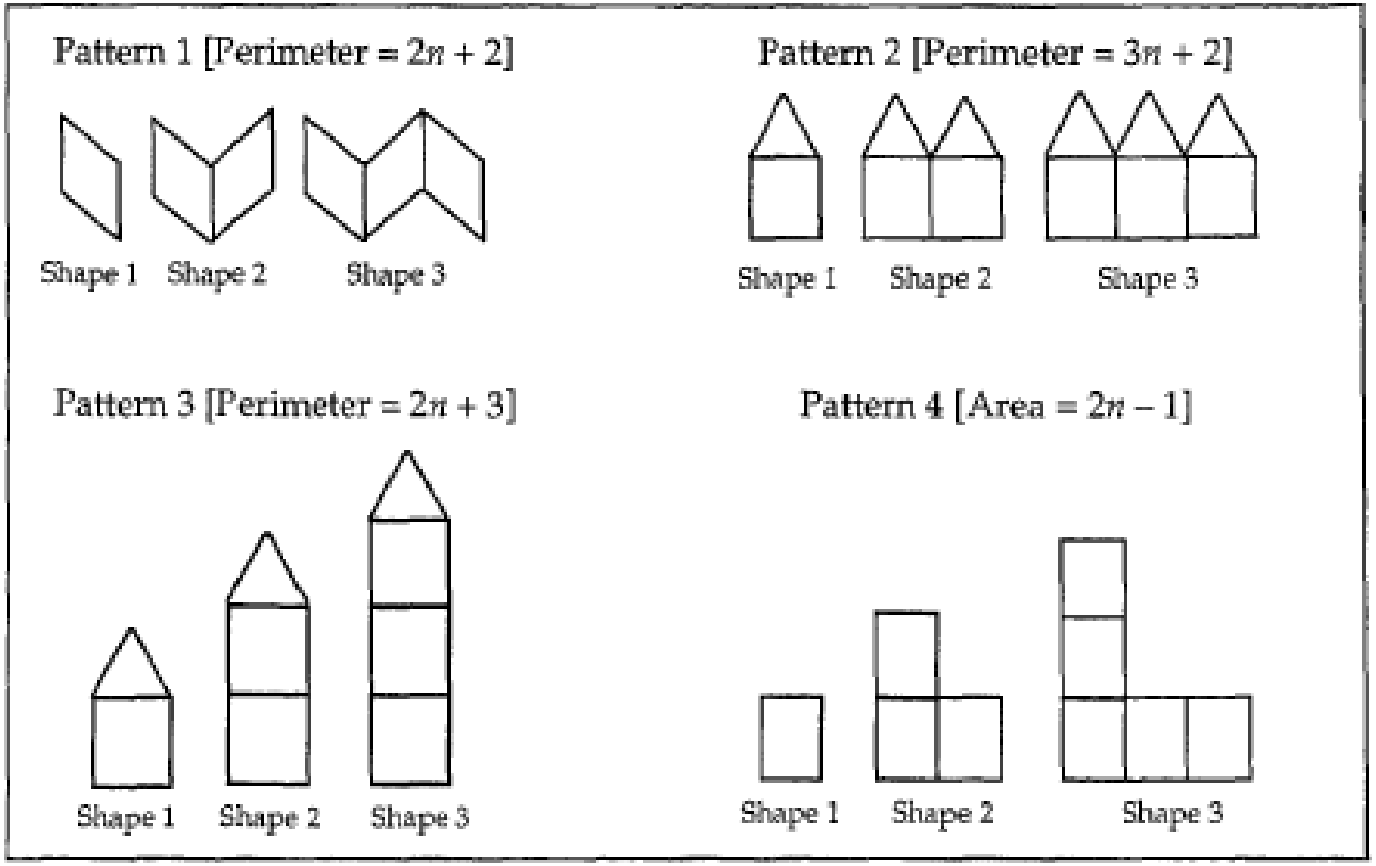


Other Pattern Problems



from Bishop, J, (2000). Linear geometric number patterns: Middle school students' strategies. *Mathematics Education Research Journal, 12*(2), 107-126.

Summary

- What are you walking away with?

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- Learn how to use student representations
- Interpret student reasoning
- Determine the type of instruction that will be best suited for a particular type of learner

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