Deducing Formulas from Sequences

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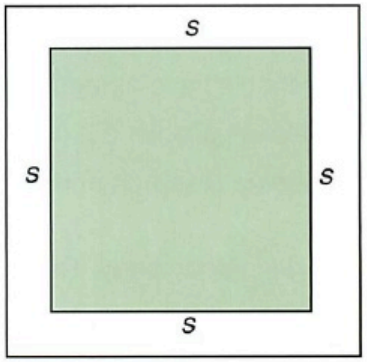
What Do You See?

Tiling Tubs

Name _____

Hot tubs and in-ground swimming pools are sometimes surrounded by borders of tiles. This drawing shows a square hot tub with sides of length *s* feet. This tub is surrounded by a border of square tiles. Each border tile measures 1 foot on each side.



1. How many 1-foot square tiles will be needed for the border of a square hot tub that has edge length *s* feet?_____

2. Express the total number of tiles in as many ways as you can. _____

Friel, S., Rachlin, S., Doyle, D., Nygard, C., Pugalee, D., & Ellis, M. (2001). Navigating through Algebra in Grades 6-8. Reston, VA: National Council of Teachers of Mathematics, 63-64.

What Do You See?

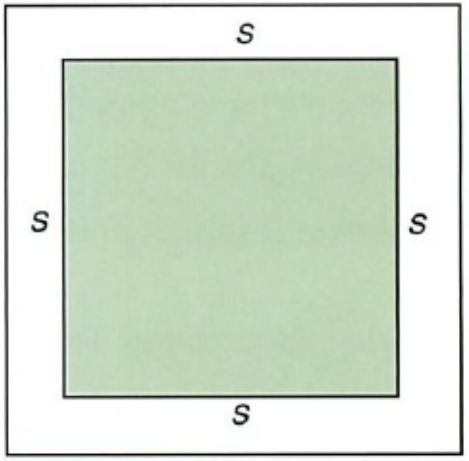
$4s + 4$

$2(s + 2) + 2s$

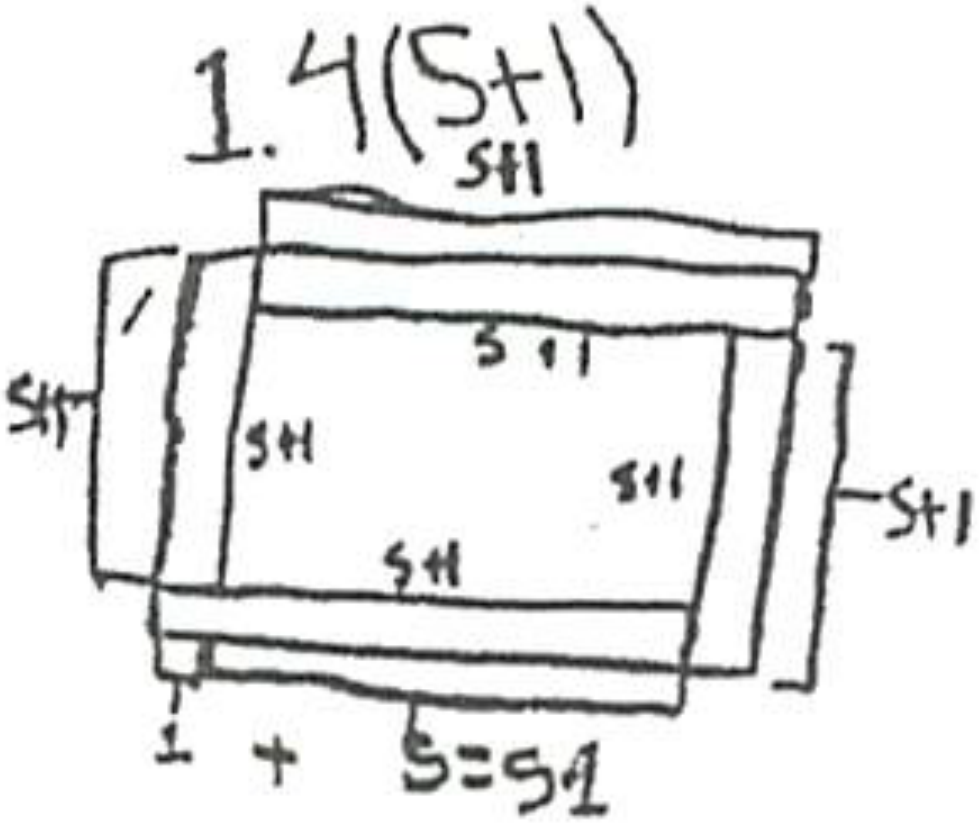
$4 (s + 2) – 4$

$4(s + 1)$

$(s + 2)^2 – s^2$

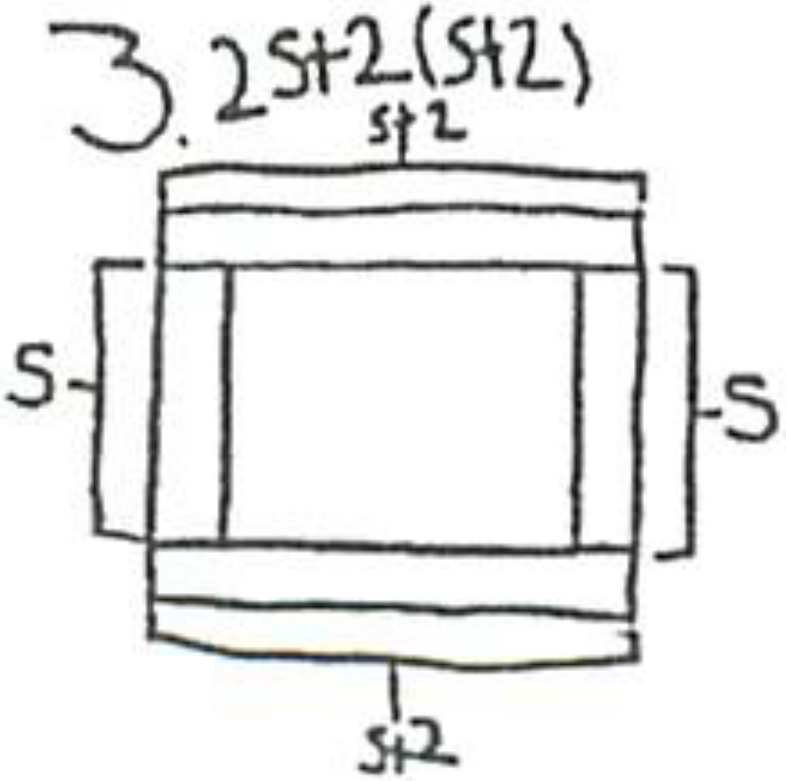


Student Work 1

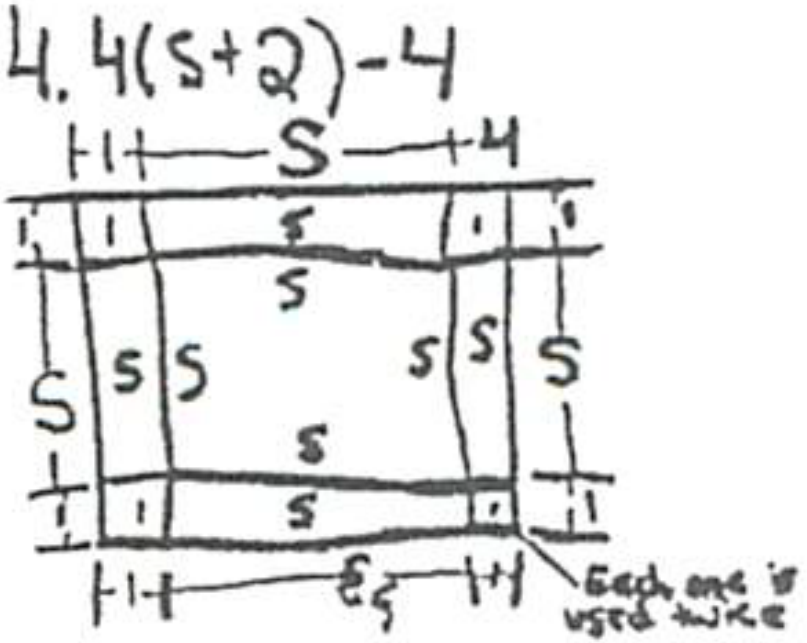


Student Work 2

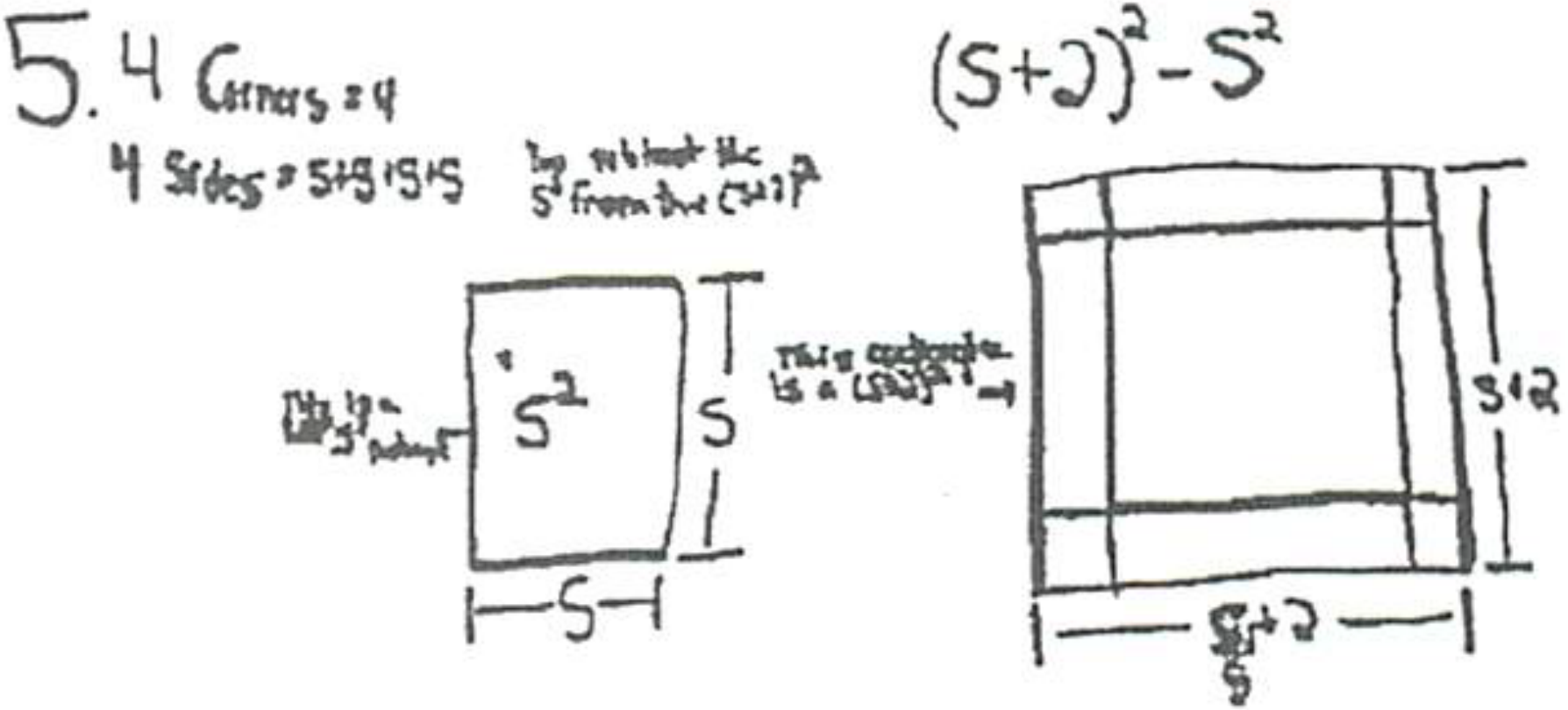
Student Work 3



Student Work 4



Student Work 5



Interpreting Expressions

1. Write algebraic expressions for each of the following:

   a. Multiply n by 5 then add 4. .....

   b. Add 4 to n then multiply by 5. .....

   c. Add 4 to n then divide by 5. .....

   d. Multiply n by n then multiply by 3. .....

   e. Multiply n by 3 then square the result. .....

http://map.mathshell.org/materials/lessons.php?taskid=221&subpage=concept

Interpreting Expressions

$$2(n + 3) = 2n + 3$$

$$\frac{10n - 5}{5} = 2n - 1$$

$$(5n)^2 = 5n^2$$

$$(n + 3)^2 = n^2 + 3^2 = n^2 + 9$$

http://map.mathshell.org/materials/lessons.php?taskid=221&subpage=concept

Matching Expressions and Words

$4(n+2)$	Multiply n by two, then add four.
$2(n+4)$	Add four to n, then multiply by two.
$4n+2$	Add two to n, then multiply by four.

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Matching Activity A/B

Card Set A: *Expressions*

E1 $\frac{n+6}{2}$	**E2** $3n^2$
E3 $2n+12$	**E4** $2n+6$
E5 $2(n+3)$	**E6** $\frac{n}{2}+6$
E7 $(3n)^2$	**E8** $(n+6)^2$
E9 $n^2+12n+36$	**E10** $3+\frac{n}{2}$
E11 n^2+6	**E12** n^2+6^2
E13	**E14**

Card Set B: *Words*

W1 Multiply ***n*** by two, then add six.	**W2** Multiply ***n*** by three, then square the answer.
W3 Add six to ***n*** then multiply by two.	**W4** Add six to ***n*** then divide by two.
W5 Add three to ***n*** then multiply by two.	**W6** Add six to ***n*** then square the answer.
W7 Multiply ***n*** by two then add twelve.	**W8** Divide ***n*** by two then add six.
W9 Square ***n***, then add six	**W10** Square ***n***, then multiply by nine
W11	**W12**
W13	**W14**

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Matching Activity A/B Key

Expressions	Words
E1	W4
E2	
E3	W7
E4	W1
E5	W5
E6	W8
E7	W2
E8	W6
E9	W6
E10	
E11	W9
E12	
	W3
	W10

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Matching Activity C/D

Card Set C: *Tables*

T1

n	1	2	3	4
Ans	14	16	18	20

T2

n	1	2	3	4
Ans			81	144

T3

n	1	2	3	4
Ans		10	15	22

T4

n	1	2	3	4
Ans	3		27	48

T5

n	1	2	3	4
Ans			81	100

T6

n	1	2	3	4
Ans		10	12	14

T7

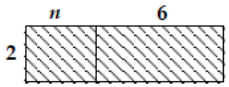
n	1	2	3	4
Ans		4		5

T8

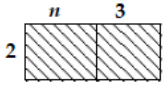
n	1	2	3	4
Ans	6.5	7	7.5	8

Card Set D: *Areas*

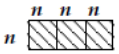
A1



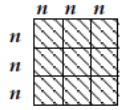
A2



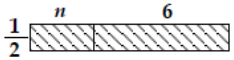
A3



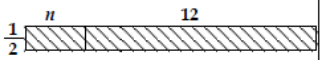
A4



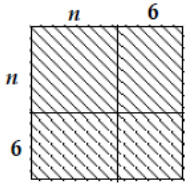
A5



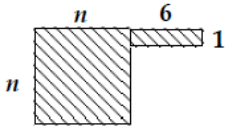
A6



A7



A8



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Matching Activity Key

Expressions	Words	Tables	Areas
E1	W4		A5
E2		T4	A3
E3	W7	T1	A1
E4	W1	T6	A2
E5	W5		A2
E6	W8	T8	A6
E7	W2	T2	A4
E8	W6	T5	A7
E9	W6		A7
E10		T7	A5
E11	W9	T3	A8
E12			
	W3		
	W10		

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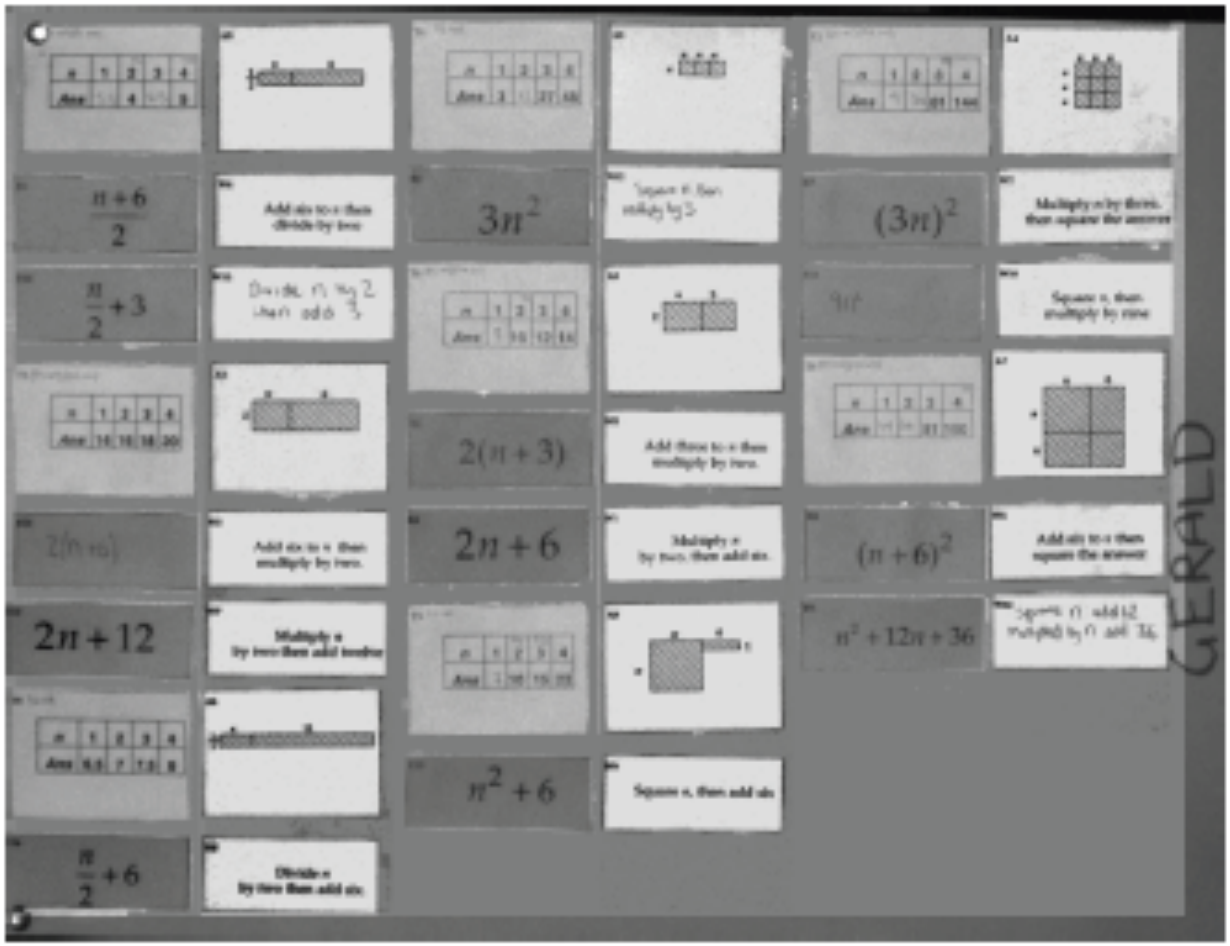
Matching Activity Poster



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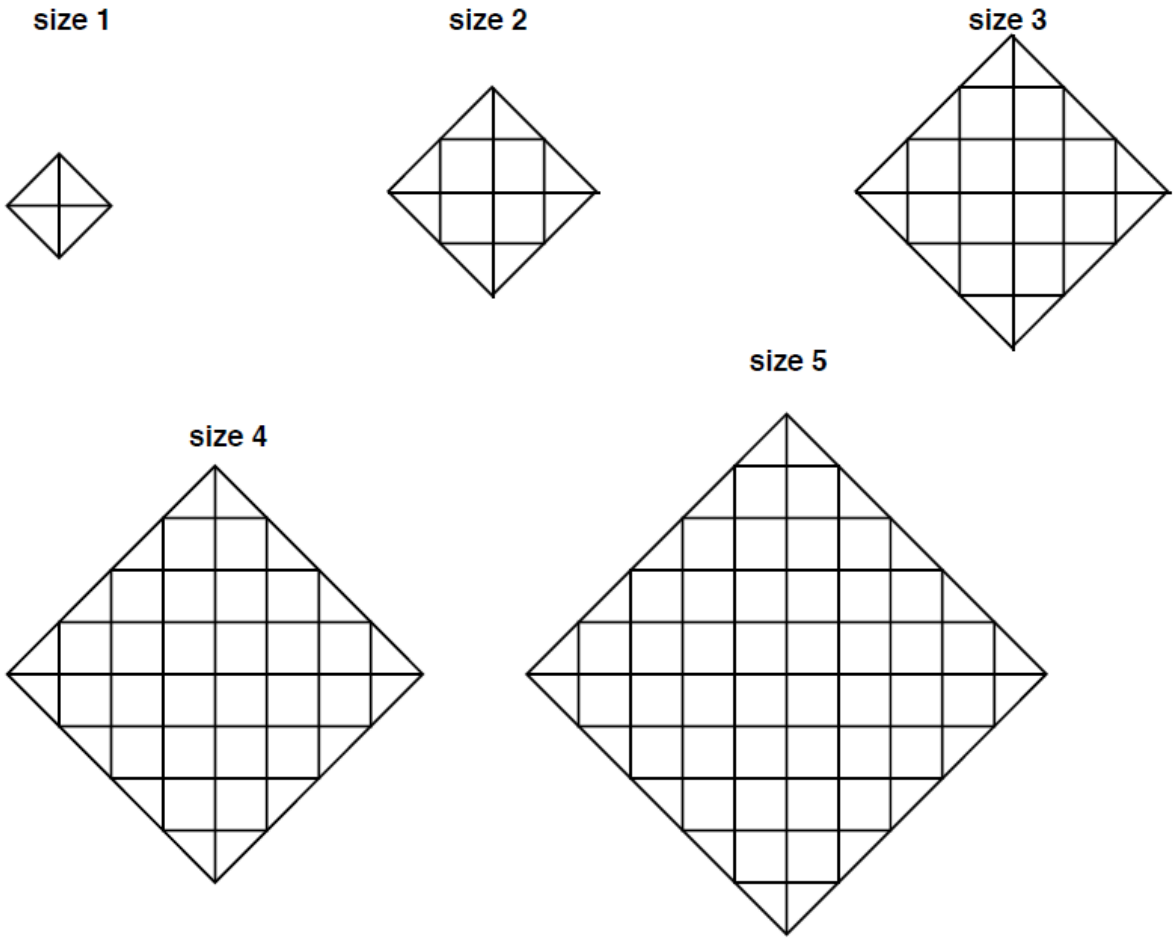
Patchwork



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Patchwork Student Work #1a

1. Complete this table to show how many triangles and squares she needs for each of these five sizes.

Size (*n*)	Number of triangles (*t*)	Number of squares (*s*)
1	4	0
2	8	4
3	12	12
4	16	24
5	20	40

2. Find a rule, or a formula, that will help Kate figure out the number of triangles she needs for cushions of different sizes. Explain how you figured it out.

For every new size you increase the triangles by 4 because 4x1=4 and 4x2=8 and so on. 4n=t You're adding four because that's the way she divided it up. (4, 8, 12, 16, 20, 24) The next size of triangles.
4 4 4 4 4

http://map.mathshell.org/materials/tasks.php?taskid=251&subpage=apprentice

Patchwork Student Work #1b

3. Use the number patterns in the table to find a rule, or a formula, that will help Kate figure out the number of squares she needs for cushions of different sizes. Explain why your rule works.

For every new size you add the sum of the triangle and square of the last size. that she did.

4 + 0 = 4

#TRIS ↑ #sqs. ↑ new #sqs.

#TRIS → 8 + 4 = 12

#sqs ↑ new # of sqs.

4. Kate has a cushion made with 180 squares. How many triangles are in this cushion? Show how you found the number of triangles.

new # sqs

20 + 40 = 60

+4

24 + 60 = 84

+4

28 + 84 = 112

+4

32 + 112 = 144

+4

36 + 144 = 180

+4

40 + 180

40

TRIANGLES = 40

Squares = 180

http://map.mathshell.org/materials/tasks.php?taskid=251&subpage=apprentice

Patchwork Student Work #2a

1. Complete this table to show how many triangles and squares she needs for each of these five sizes?

Size *(n)*	Number of triangles *(t)*	Number of squares *(s)*
1	4	0
2	8	4
3	12	12
4	16	24
5	20	40

2. Find a rule, or a formula, that will help Kate figure out the number of triangles that she needs for cushions of different sizes. Explain how you figured it out.

y = 4x
y- the # of triangles increases by 4 every time

http://map.mathshell.org/materials/tasks.php?taskid=251&subpage=apprentice

Patchwork Student Work #2b

3. Use the number patterns in the table to find a rule, or a formula, that will help Kate figure out the number of squares she needs for cushions of different sizes. Explain why your rule works.

$y = 2x^2 + -2x$

- it's quadratic ($y = ax^2 + bx + 0$)

$y = 2x^2 - 2x$

$y = 2x(x - 1)$

4. Kate has a cushion made with 180 squares.
How many triangles are in this cushion?
Show how you found the number of triangles.

$180 = 2x(x - 1$

$= 2 \times 20(19)$

$180 = 2.10(10 - 1)$ $x = 10$

$= 180$ triangles $4x = 40$

http://map.mathshell.org/materials/tasks.php?taskid=251&subpage=apprentice

Summary

- What are you taking away from this workshop?

- What questions do you have?