


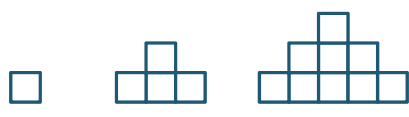
How Do Algebra 1 Students See Patterns?

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<http://nolanmath.com/Patterns.htm>




Setting the Stage

In the figure, as the step changes,
 _____ also changes.



Step 1 Step 2 Step 3

Peterson, Blake E. "Linear and Quadratic Change: A Problem from Japan," *Mathematics Teacher*, NCTM: Reston, VA, October 2006. Pages 206-212.



Setting the Stage Possibilities

The following list gives some examples of student responses:

perimeter	number of nonadjacent squares
height	number of right angles
width	sum of the interior angles
size of enclosing rectangle	number of diagonals
number of "toothpicks"	leftover space
number of interior toothpicks	number of segments
number of intersections	number of parallel lines
number of corners	length of longest line
number of convex corners	number of rectangles
number of squares	

Peterson, Blake E. "Linear and Quadratic Change: A Problem from Japan," *Mathematics Teacher*, NCTM: Reston, VA, October 2006. Pages 206-212.

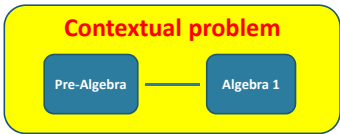
Goals

- Offer activities with multiple access points for students
- Consider multiple student representations
- Interpret student reasoning
- Determine the type of instruction that will be best suited for a particular type of learner

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Access Points

We will look at some problems that have multiple “access points.” Teachers and students can enter at various points of the problem based on their background and readiness.



The diagram shows a yellow rounded rectangle labeled "Contextual problem" in red. Inside, there are two blue rounded rectangles: "Pre-Algebra" on the left and "Algebra 1" on the right, connected by a horizontal line.

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Representational Fluency

We need to assist our students in moving among representations:

- Verbal
- Concrete
- Numeric
- Geometric
- Algebraic


to **generalize** strategies and solve problems.

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Marcy's Dots


A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues infinitely.

(1st step)




2 dots

(2nd step)



6 dots

(3rd step)



12 dots

Marcy has to determine the number of dots in the 20th step, but she does not want to draw all 20 pictures and then count the dots.

Explain and show how she could do this and give the answer that Marcy should get for number of dots.

Item from 1992 NAEP test 7

Content: Expressions and Equations

Grade 6

Apply and extend previous understandings of arithmetic to algebraic expressions.

Reason about and solve one-variable equations and inequalities.

Represent and analyze quantitative relationships between dependent and independent variables.

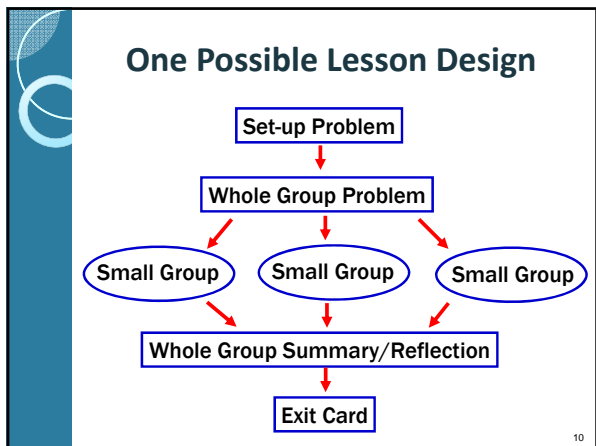
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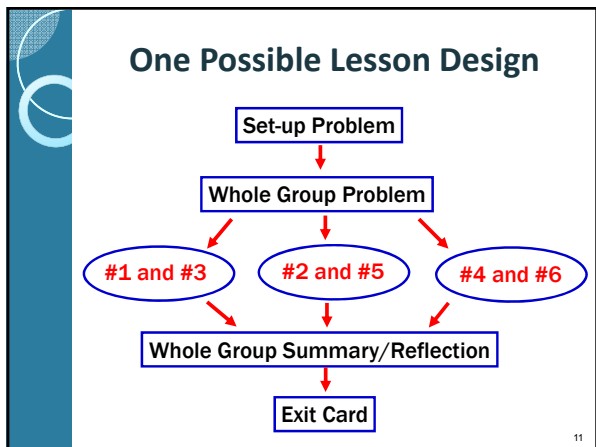
With which practice(s) were we engaged?

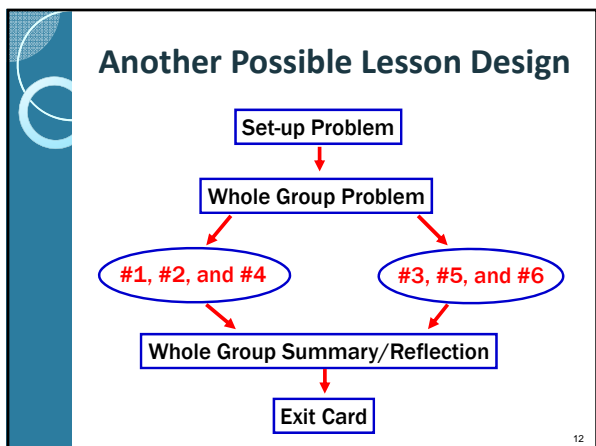
The 8 Standards for Mathematical Practice:

- 1 Make sense of problems and persevere in solving them
- 2 Reason abstractly and quantitatively
- 3 Construct viable arguments and critique the reasoning of others
- 4 Model with mathematics
- 5 Use appropriate tools strategically
- 6 Attend to precision
- 7 Look for and make use of structure
- 8 Look for and express regularity in repeated reasoning

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Other Pattern Problems

Pattern 1 [Perimeter = $2n + 2$]

Shape 1 Shape 2 Shape 3

Pattern 2 [Perimeter = $3n + 2$]

Shape 1 Shape 2 Shape 3

Pattern 3 [Perimeter = $2n + 3$]

Shape 1 Shape 2 Shape 3

Pattern 4 [Area = $2n - 1$]

Shape 1 Shape 2 Shape 3

Bishop, J. (2000). Linear geometric number patterns: Middle school students' strategies. *Mathematics Education Research Journal*, 12(2), 107-126.

Predict how many pennies fit in a 22-inch circle

<http://blog.mrmeyer.com/>

Revisiting the Goals

- Offer activities with multiple access points for students
- Consider multiple student representations
- Interpret student reasoning
- Determine the type of instruction that will be best suited for a particular type of learner
