Represent and Analyze Linear and Nonlinear Functions including Simultaneous Functions

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July 2013

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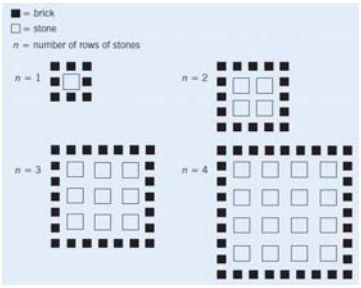
Patio Pattern Task

■ = brick
□ = stone
n = number of rows of stones



Source: You, Z. (2007/2008). Take time for action: Investigating students' thinking about functional relationships. *Mathematics teaching in the middle school, 13* (5), 312-315.

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Common Core Standards

Analyze and solve linear equations and pairs of simultaneous linear equations.

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  - CCSS.Math.Content.8.EE.C.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Define, evaluate, and compare functions.

- CCSS.Math.Content.8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

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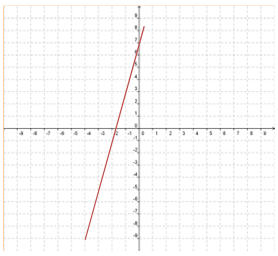
How do you make comparisons between an equation and a graph?

$y = -4x + 5$

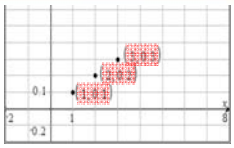


from LEARNZILLION

003010_8.F.2_Compare_two_functions_by_analy-slides

Compare Equation and Graph

$y = 0.05x$



- Function
- One-to-one
- Constant rate of change = 0.05

- Function
- One-to-one
- Constant rate of change = 0.10

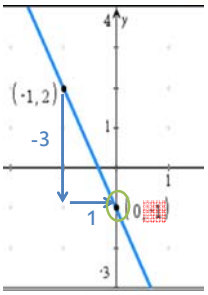
from LEARNZILLION

Compare Equation and Graph

$y = \underline{-3}x + \underline{-1}$

Rate of change

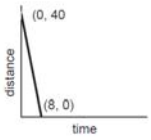
y – intercept



from LEARNZILLION

Using Graphs to Introduce Functions

1. Consider the graph at the right, where time in seconds is graphed along the x-axis and distance in feet is graphed along the y-axis. The graph shows a girl's distance from a spaceship ride as a function of time. A table of values that could accompany the graph is also given. Use the table to help you decide which equation matches the graph.



Time	Distance
0	40
2	30
4	20
6	10
8	0

a) $y = 40x - 8$ *b*) $y = 40 - 8x$ *c*) $y = 5x + 40$ *d*) $y = 40 - 5x$

Van Dyke, F. (2003). Using Graphs to Introduce Functions. *Mathematics Teacher, 96*(2), 126-137.

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PARCC Sample Item

A portion of the graph of a quadratic function $f(x)$ is shown in the xy - plane. Selected values of a linear function $g(x)$ are shown in the table.

x	$g(x)$
-4	7
-1	1
2	-5
5	-11

For each comparison below, use the drop-down menu to select a symbol that correctly indicates the realtionship betweeen the first and the second quantity.

First Quantity	Comparison	Second Quantity
The y-coordinate of the y-intercept $f(x)$	↕	The y-coordinate of the y-intercept $g(x)$
$f(3)$	↕	$g(3)$
Maximum value of $f(x)$ on the interval $-5 \le x \le 5$	↕	Maximum value of $g(x)$ on the interval $-5 \le x \le 5$
$\frac{f(5) - f(2)}{5 - 2}$	↕	$\frac{g(5) - g(2)}{5 - 2}$

http://parcconline.org/samples/mathematics/high-school-functions

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Keeping Trains on the Tracks

GRAPHING TRAIN TRAVEL

Liza Dash lives in Philadelphia and has to get to Atlanta as fast as possible. Since Liza has a fear of flying, she plans to travel by train. One train leaves at 12:00 noon and travels an average of 70 mph. A second train leaves 2 hours later but averages 85 mph. Assuming that the distance between the two cities is about 750 miles, which train should she take to arrive in Atlanta as early as possible?

1. If you average 70 mph, how many hours will it take to travel at least 750 miles?

2. Fill in the table with the distance traveled by each train at the end of each hour.

Time	Hours	Train 1	Train 2
12:00 a.m.	0		
1:00 p.m.	1		
2:00 p.m.	2		
3:00 p.m.	3		
4:00 p.m.	4		
5:00 p.m.	5		
6:00 p.m.	6		
7:00 p.m.	7		
8:00 p.m.	8		
9:00 p.m.	9		
10:00 p.m.	10		
11:00 p.m.	11		

Moyer, T. O. (2010). Mathematical explorations: Keeping all the trains on the tracks. *Mathematics Teaching in the Middle School, 16*(2), 116-120.

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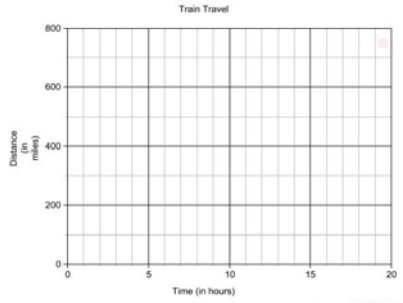
Keeping Trains on the Tracks



Made with "Create a Graph" at http://nces.ed.gov/nceskids/createagraph/default.aspx

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Keeping Trains on the Tracks

- Using your graphing calculator and Y1 and Y2 to represent each train, graph the movement of each train from Philadelphia to Atlanta.
- Describe the window settings you have chosen, and explain how they relate to the problem.

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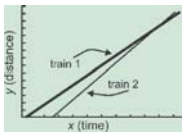
Keeping Trains on the Tracks

```
WINDOW
 Xmin=0
 Xmax=11
 Xscl=1
 Ymin=0
 Ymax=750
 Yscl=50
 Xres=1
```



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Keeping Trains on the Tracks

```
NORMAL  SCI  ENG
FLOAT  0 1 2 3 4 5 6 7 8 9
RADIAN  DEGREE
FUNC  PAR  POL  SEQ
CONNECTED  DOT
SEQUENTIAL  SIMUL
REAL  a+bi  re^θi
      HORIZ  G-T
        ↓NEXT↓
```

```
Plot1 Plot2 Plot3
\X1T■70T
 Y1T■1
\X2T■85(T-2)
 Y2T■2
\X3T=
 Y3T=
\X4T=
```

```
WINDOW
 Tmin=0
 Tmax=11
 Tstep=1
 Xmin=0
 Xmax=750
 Xscl=50
↓Ymin=■
```

```
WINDOW
↑Tstep=1
 Xmin=0
 Xmax=750
 Xscl=50
 Ymin=0
 Ymax=3
 Yscl=1
```

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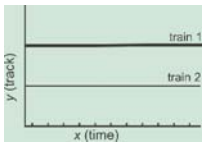
Keeping Trains on the Tracks



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Classify Systems of Equations

x	-3	2	3
y	-3	7	9

A

x	0	2	4
y	5	7	9

B

x	-1	0	2
y	5	1	7

C

x	-1	0	2
y	1	3	7

D

1a. Which of these tables of values satisfy the equation $y = 2x + 3$? Explain how you checked.

Classifying Solutions to Systems of Equations,
http://map.mathshell.org/materials/lessons.php?taskid=411&subpage=concept

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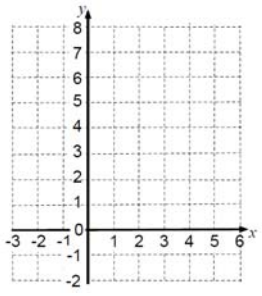
Classify Systems of Equations



$y = 2x + 3$

x	-2	0	
y			5

$x = 1 - 2y$

x	0		5
y		0	

Classifying Solutions to Systems of Equations,
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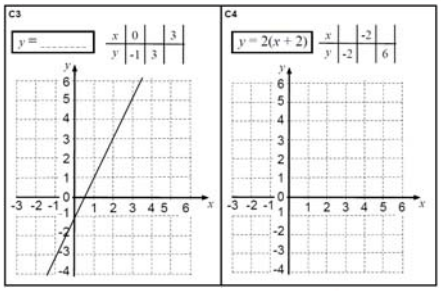

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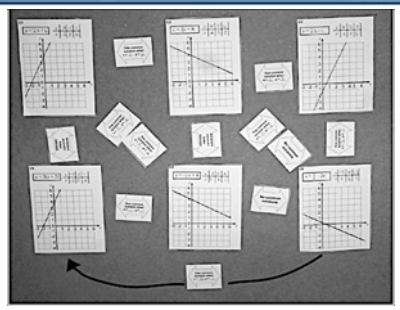

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Concept Map: Systems of Equations

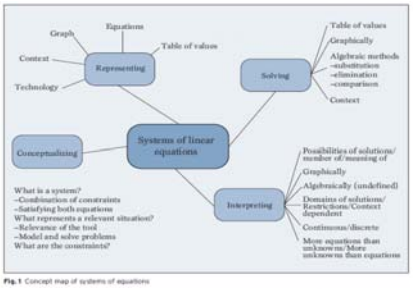

Fig. 1 Concept map of systems of equations

Proulx, J., Beisiegel, M., Miranda, H., & Simmt, E. (2009). Rethinking the teaching of systems of equations. *Mathematics Teacher, 102*(7), 526-533.

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Solve systems of equations.

- CCSS.Math.Content.HSA-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- CCSS.Math.Content.HSA-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- CCSS.Math.Content.HSA-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.
- CCSS.Math.Content.HSA-REI.C.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.
- CCSS.Math.Content.HSA-REI.C.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

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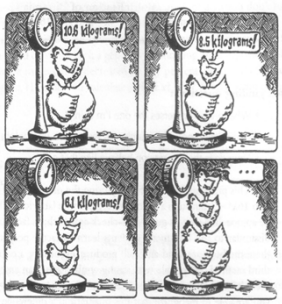
Extra, Extra



Driscoll, Mark. Fostering Algebraic Thinking, A Guide for Teachers Grades 6-10. Heinemann Publishing: Portsmouth, Hew Hampshire, 1999.

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Wrap Up

Name something new that you feel you are leaving with a good understanding.

Name something that is going round and round in your head.

Name something that squared with your thinking.

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