

Activities

Using Graphs to Introduce Functions

Students may resort to complicated algebraic expressions rather than read information from a graph

THE PAST DECADE HAS SEEN A SHIFT TOWARD FUNCTIONS as a central theme in beginning algebra. The advent of graphing calculators has meant that the graphical representation of functions is accessible and can be used in a meaningful way. Yet evidence indicates that students ignore graphs and resort to complicated algebraic expressions rather than read information from a graph. In “Understanding Connections between Equations and Graphs,” Eric Knuth (2000) reported on a study that he conducted with 178 students from a suburban high school. He concluded that students may be missing the basic “Cartesian connection” and that they do not recognize that a graph and an equation are two representations for the same set of points.

One idea for linking the representations and highlighting the importance of graphs is to introduce the concept of function using graphs, or pictorial representations of functions. A natural progression exists from qualitative graphs to quantitative graphs to tables to equations. Students become comfortable with functions that are introduced using this progression.

This article presents four activities that use this progression to introduce the theme of distance from an object as a function of time. The advantage of this application is that students are familiar with the idea, can walk the motion, and can use motion detectors to produce similar graphs. The disadvantage is that students may not understand the dependent variable and may think that it is speed or distance traveled, rather than distance from an object. They also may have a tendency to use iconic translation, that is, to impose an image of the scene onto the graph. An application in which these prob-

lems are avoided is one in which the value of the variable changes as a function of time, and teachers are encouraged to make up similar activities. After students have completed **sheet 1**, the teacher may want to discuss the answers in class. The class as a whole might discuss what each graph reveals about motion to underline the meaning of the dependent variable and to prevent students from using iconic translation.

Concentrating first on qualitative graphs allows students to think on an abstract level and to look at graphs globally. Exercises that connect the classroom with the real world may appeal to students who do not consider themselves mathematically oriented. By comparing qualitative graphs, a teacher can introduce important concepts from basic algebra without the burden of algebraic notation. The introduction of coordinates and points on the graphs allows students to interpolate and give specific information about the situation. The transition to tables is then not at all difficult. Finally, when students are comfortable with the graphical and tabular representations, the teacher can introduce equations.

These activities can be used for grade levels 8–14. The activities challenge students who have a prealgebra background and are a good review for those who have had basic algebra. The only materials needed to complete the activity are the activity

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This section is designed to provide in reproducible formats mathematics activities appropriate for students in grades 7–12. This material may be reproduced by classroom teachers for use in their own classes. Readers who have developed successful classroom activities are encouraged to submit manuscripts, in a format similar to the “Activities” already published, to the senior journal editor for review. Of particular interest are activities focusing on the Council’s curriculum standards, its expanded concept of basic skills, problem solving and applications, and the uses of calculators and computers.

Write to NCTM, attention: infocentral, or send e-mail to infocentral@nctm.org, for the catalog of educational materials, which lists compilations of “Activities” in bound form.—Ed.

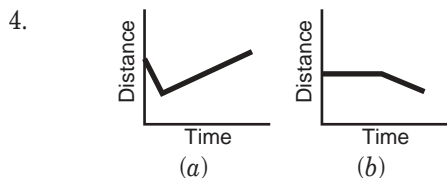
sheets and blank paper for answers. A graphing calculator and a motion detector may also be used to complement the activity. The objective is to help students deepen their understanding of functions, graphs, and the underlying equivalence of the numeric, tabular, and algebraic representations of functions. Students can complete the activity in groups. Each student can keep his or her own record of answers and can make up situations and graphs that differ from those created by others in the group.

Each student should keep a copy of the four activity sheets, complete with answers, so that the teacher can assess their work. The teacher should discuss each sheet when students have completed it. After students complete all four sheets, the teacher leads the class in a discussion of the notion of a function as a relationship between quantities that change. The activity sheets focus on different ways of representing this relationship. After completing the activity sheets, students write a paragraph on the aspect of the relationship that each representation highlights. Looking at the equation reveals initial distance, rate, and direction. A table gives actual times with corresponding distances, whereas a graph is best for comparing two different motions. This progression can be developed for other functions—both linear and nonlinear—that students typically study.

SOLUTIONS

Sheet 1

- The correct answer is (b). A student who thinks of the dependent variable as speed might give (a) as the answer, and a student who thinks of the dependent variable as distance covered might think that (c) is the answer.
- Answers will vary. For graph (a), Hugo might be standing a fixed distance from the object or moving in a circle around the object.
- The correct answer is (c). A student who thinks that the roller coaster somehow gets pictured in the graph might give (a) as the answer; and if iconic translation occurs, that is, if the image of Mary, who is standing still, imposes itself on the graph, a person might give (d) as the answer.



The sharper slope in the part of (a) nearest the vertical axis indicates that Chris's walk toward the tunnel was faster than his walk away from the tunnel.

- Teachers may want to have students work on part (a) first, go over the answer, and then try parts (b), (c), and (d). Answers should indicate whether the people are moving toward or away from the ride, in addition to indicating the person who is moving faster. Sample answers are as follows.

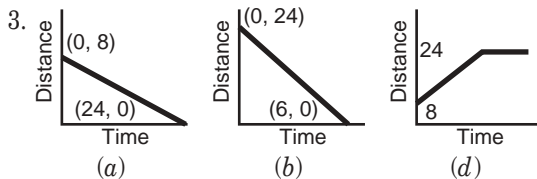
- Initially, Al was about twice as far from the ride as Bea was, and he took about one-quarter of the amount of time that she did to arrive at the ride. Al was therefore moving eight times as fast as Bea.
- Initially, Clay was about four times as far from the ride as Di. Di moved about one-half the distance that Clay moved in three times the time, so she was moving at six times his rate. Di was moving away from the ride and Clay was moving toward it.
- Eli was initially about three times as far from the ride as Faye was. They both reached the ride at the same time, so Eli was traveling three times as fast as Faye.
- Hal was initially about half as far as Gus was from the ride. Hal moved away, and Gus stood still.

Although we can compare the two motions of the members of the pairs, we have no idea how far the people were from the rides nor how fast they moved, since the graphs do not indicate units. Al, Clay, Eli, and Hal were all walking faster than Bea, Di, Faye, and Gus, respectively. The starting point, or origin, of the walk for both is the ride. The x -intercepts indicate time taken for the walk; the y -intercepts indicate initial distance from the ride.

- Jane's walk is depicted in (a), Angela's in (c), and Kathy's in (b). In part (a), equal distances are covered in equal time. As time passes in (b), less distance is covered in equal time intervals; whereas a greater distance is covered in (c) as time passes.

Sheet 2

- The correct answer is (c).
- Alisa moved 24 feet in all.
 - She took 8 seconds to walk this distance.
 - Alisa walked at the rate of 3 feet per second.
 - After 1 second, she was 21 feet away from the pinball machine area.
 - After 2 seconds, she was 18 feet away.
 - After 5 seconds, she was 9 feet away.



The graphs do not indicate units

4. The correct answer is (c).
5. a) Hugo was 9 feet from a hot-dog stand and inched steadily toward it, reaching it after 21 seconds. He moved at the rate of $\frac{3}{7}$ a foot per second.
- b) Hugo was 3 feet from a hot-dog stand and moved steadily away from it. After 9 seconds, he was 21 feet away. He moved at a rate of 2 feet per second.
- d) Hugo waited for 3 seconds, then moved away at a rate of 3.5 feet per second. After 9 seconds, he was 21 feet away from the hot-dog stand.
6. a) The person started 15 meters from the exit and moved toward it at a rate of 3 meters per second, reaching it after 5 seconds. He then waited at the exit for 6 seconds and moved away at a rate of 2 meters per second. Since no indication of direction is given, he or she could be leaving the park, entering the park, or approaching and then backing away from the exit.
- b) The individual started 9 meters from the exit, then moved away at a rate of 1.5 meters per second for 4 seconds. He then returned to the exit at a rate of 5 meters per second.

Sheet 3

1. The answer is table (b). It is the only table that shows a constant rate of change.
2. The correct answer is (c).

3. a)

Time	Distance
0	0
1	6
2	12
3	18
4	24
5	30
6	36

b)

Time	Distance
0	42
1	35
2	28
3	21
4	14
5	7
6	0

d)

Time	Distance
0	0
1	7
2	14
3	21
4	28
5	35
6	42

4. Entries in the tables for problem 4 on sheet 2 could be as follows:

a)

Time	Distance
0	9
7	6
14	3
21	0

b)

Time	Distance
0	3
1	5
2	7
3	9
4	11
⋮	⋮
9	21

c)

Time	Distance
0	9
1	13
2	17
3	21

d)

Time	Distance
0	0
1	0
2	0
3	0
4	3.5
5	7
6	10.5
7	14
8	17.5
9	21

5. For problem 6 on sheet 2, the tables are as follows:

a)

Time	Distance
0	15
2	9
3	6
5	0
6	0
⋮	⋮
11	0
12	2
13	4
14	6
15	8
16	10
17	12

b)

Time	Distance
0	9
1	10.5
2	12
3	13.5
4	15
5	10
6	5
7	0

Sheet 4

1. The correct equation is (d). Each point in the table satisfies the equation. The initial distance is the y -intercept on the graph. In the table, the initial distance is the line with 0 as the time entry. In the equation, the initial distance the constant term 40, which corresponds to the y -intercept.

2. a) The equation (a) $y = 42 - 6x$ corresponds to the table given.

b) $y = 42 - 7x$ corresponds to table (b).

c) $y = 6x$ corresponds to table (a).

d) $y = 7x$ corresponds to table (d).

3. The correct answer is (b). Moving 20 feet in 8 seconds implies a rate of 2.5 feet per second, which is the coefficient of the variable standing for seconds elapsed. The initial distance corresponds to the constant term in the equation.

The answers will vary. For (a), one possibility is, "Zarah was 4 feet from a stand and walked away at 6 feet per second." For (c), one possibility is, "Zarah was 24 feet from a stand and walked away at 4 feet per second." For (d), one possibility is, "Zarah was 8 feet from a stand and walked away at 6 feet per second."

4. The correct answer is (d). The rate of 3 feet per second is the absolute value of the coefficient of the variable standing for seconds elapsed. The coefficient is negative when Josh is walking toward the bumper cars, so his distance from them decreases as time passes. The initial distance of 30 feet corresponds to the constant term in the equation.

For (a), one possibility is, "A man in a car was 3 feet from a sign, and he rode away at a rate of 30 feet per second." For (b), one possibility is, "An object was 3 feet above sea level and dropped down at a rate of 30 feet per second." For (c), one possibility is, "A man was 30 feet from a sign and walked away at a rate of 3 feet per second."

5. The rate of motion is positive when the individual is moving away from the object, and it is negative when he or she is moving toward the object. The distance is increasing (positive) when the individual is moving away from the object, and it is decreasing (negative) when the individual is moving toward the object. The rate of motion is the slope of the line on the graph. The rate of motion can be determined in the table by the difference in distance for each increment in time, that is, difference in distance divided by difference in time.

6. a) $y = 9 - (3/7)x$

b) $y = 3 + 2x$

c) $y = 9 + 4x$

d) $y = -10.5 + (7/2)x$

7. 6(a) on sheet 2:

$$y = 15 - 3x, \text{ if } 0 < x \leq 5$$

$$y = 0, \text{ if } 5 < x \leq 11$$

$$y = -22 + 2x, \text{ if } 11 < x \leq 17$$

6(b) on sheet 2:

$$y = 9 + (3/2)x, \text{ if } 0 < x \leq 4$$

$$y = 35 - 5x, \text{ if } 4 < x \leq 7$$

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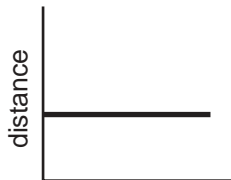
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(Worksheets begin on page 130)

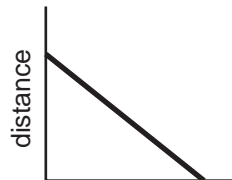
Each of the graphs on this page represents distance from an object as a function of time.

1. On the following graphs, distance, as labeled on the y -axis, refers to distance from an amusement park. Which graph best matches the following sentence?

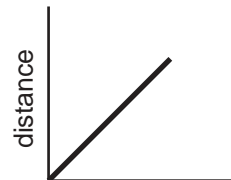
Hugo walked at a steady pace toward the amusement park.



(a)



(b)



(c)

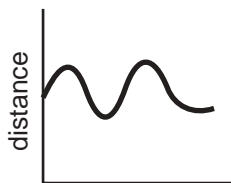


(d)

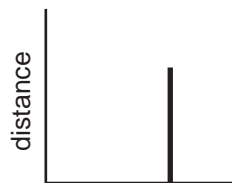
2. Describe a situation involving distance and time that could match each of the graphs that you did not choose as the answer to problem 1.

3. In this problem, distance on the y -axis stands for Mary's distance from the roller-coaster entrance. Which graph best matches the following sentence?

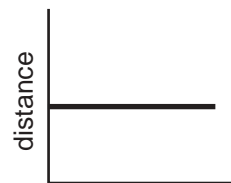
While she was standing still, Mary looked at the roller-coaster track in the distance.



(a)



(b)



(c)



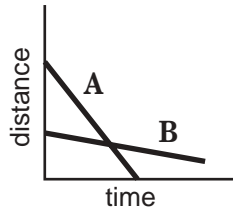
(d)

4. Draw a graph that could correspond to each of the following situations. For graph (a), let the y -axis represent distance from the tunnel of love. For graph (b), let the y -axis represent distance from the entrance to the Ferris wheel.

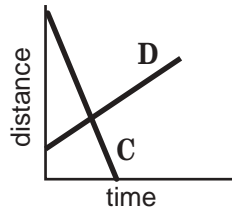
a) Chris walked quickly toward the tunnel of love and then slowly walked away from it.

b) We stood for a while, and then the line in which we were waiting began to slowly move toward the Ferris wheel.

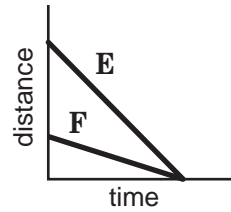
5. Each of the following graphs depicts the relationship between distance from a ride and time elapsed for two people. Each person walks at a steady rate directly toward or away from the ride or stands still. For each graph, describe the relationships and make observations and comparisons.



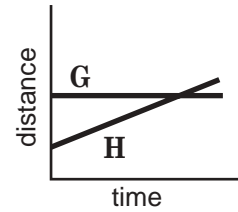
(a)



(b)



(c)



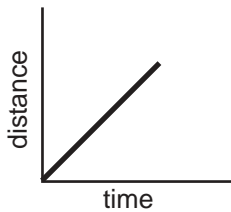
(d)

In addition to your observations, answer the following questions for each example:

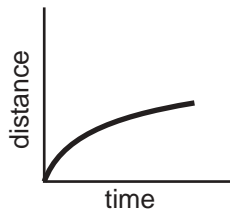
Which person is walking faster?

What is the significance of the x -intercept? The y -intercept?

6. The graphs below show motion away from the park for three different mothers. Jane moves at a steady pace, Angela speeds up as she walks away, and Kathy slows down as she moves away. Which graph matches which woman's motion? Explain your reasoning.



(a)



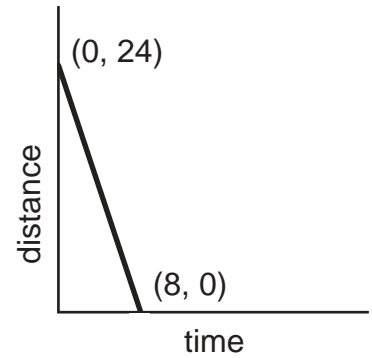
(b)



(c)

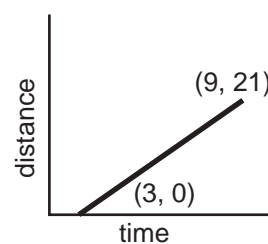
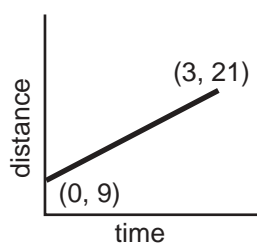
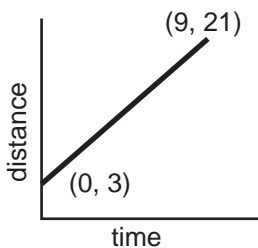
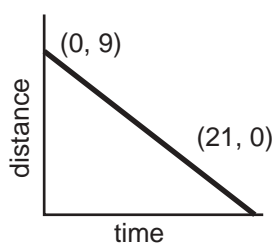
1. Consider the graph below, in which time in seconds is graphed along the x -axis and distance in feet is graphed along the y -axis. The graph shows Alisa's distance from the pinball machine area as a function of time. Which sentence is a good match for the graph?

- a) Alisa stood 8 feet from the pinball machine area and moved toward it, reaching it after 24 seconds.
- b) Alisa stood 24 feet from the pinball machine area and moved toward it at a rate of 4 feet per second.
- c) Alisa stood 24 feet from the pinball machine area and moved toward it, reaching it after 8 seconds.
- d) Alisa stood 8 feet from the pinball machine area and moved away from it, stopping when she was 24 feet away.



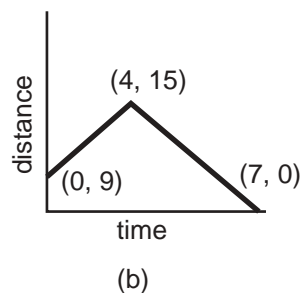
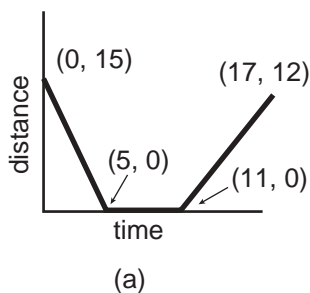
- 2. a) How many feet did Alisa move in all? _____
 - b) How long did she take to walk this distance? _____
 - c) The straight line indicates a steady pace. How many feet did Alisa walk in each second? _____
 - d) How far was Alisa from the area after 1 second? _____
 - e) How far was Alisa from the area after 2 seconds? _____
 - f) How far was Alisa from the area after 5 seconds? _____
3. For each answer not chosen in question 1, sketch a graph that could correspond to that description. Write the description next to your graph.

4. Hugo was standing 9 feet from the hot-dog stand. He walked away, and after 3 seconds he was 21 feet away. Which of the following graphs corresponds to this situation?

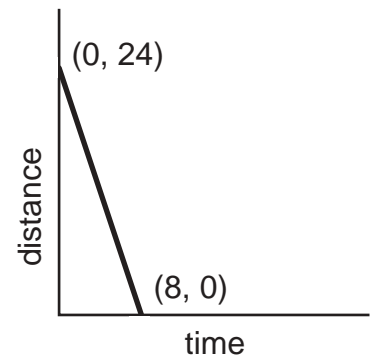


5. Write a description, similar to the one given in problem 1, that could correspond to each of the graphs not chosen in question 4. Determine the walking rate in each situation.

6. In the following two graphs, distance in meters from the main exit is graphed as a function of time in seconds for one individual. Describe the motion using the coordinates shown as end-points of the line segments. Determine the rate at which the person walks for each segment. Assume that the person is moving directly toward or away from the exit that or he or she is standing still.



1. Consider again the graph at the right, where time in seconds is graphed along the x -axis and distance in feet is graphed along the y -axis. The graph shows Alisa's distance from a pinball machine area as a function of time. Which table describes the graph? Explain your decision.



Time	Distance	Time	Distance	Time	Distance
0	24	0	24	0	24
2	16	2	18	2	18
4	8	4	12	4	11
6	0	6	6	6	3
8	0	8	0	8	0
(a)		(b)		(c)	

2. Consider the following table.

Time	Distance from Water Ride
0	42
1	36
2	30
3	24
4	18
5	12
6	6
7	0

Choose the sentence that best describes the table.

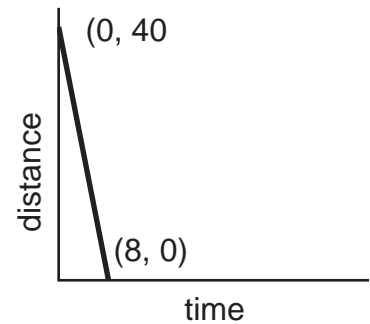
- a) Kirk walked away from the water ride at a rate of 6 feet per second.
- b) Kirk was 42 feet away from the water ride and walked toward it at a rate of 7 feet per second.
- c) Kirk was 42 feet away from the water ride and walked toward it at a rate of 6 feet per second.
- d) Kirk walked away from the water ride at a rate of 7 feet per second.

3. For each answer that did not describe the table in question 2, make a table that could correspond to it. Give entries for each second from 0 to 6.

4. Make tables that correspond to the graphs given on sheet 2, question 4.

5. Make tables that correspond to the graphs on sheet 2, question 6.

1. Consider the graph at the right, where time in seconds is graphed along the x -axis and distance in feet is graphed along the y -axis. The graph shows a girl's distance from a spaceship ride as a function of time. A table of values that could accompany the graph is also given. Use the table to help you decide which equation matches the graph.



Time	Distance
0	40
2	30
4	20
6	10
8	0

- a) $y = 40x - 8$ b) $y = 40 - 8x$ c) $y = 5x + 40$ d) $y = 40 - 5x$

Explain why you made your choice.

How is the initial distance indicated on the graph? In the table? In the equation?

2. Consider again the sentences created on sheet 3, question 2, and the tables created in question 3.

Each of the following is an equation that describes one and only one of the sentences. Here, x corresponds to time in seconds and y to distance in feet. Write the appropriate sentence under each equation. Indicate the table that goes with each equation. Does each pair of values in the tables satisfy the equation?

a) $y = 42 - 6x$

b) $y = 42 - 7x$

c) $y = 6x$

d) $y = 7x$

3. Zarah is standing 4 feet from a concession stand. She walks away at a steady pace. After 8 seconds, she is 24 feet away. Choose the linear equation that indicates her distance, y , from the stand as a function of time elapsed in seconds, x , since Zarah began walking. Explain your reasoning.

a) $y = 4 + 6x$ b) $y = 4 + 2.5x$ c) $y = 24 + 4x$ d) $y = 8 + 6x$

For each answer not chosen, give a scenario that describes the equation.

4. Josh is 30 feet from the bumper cars. He walks toward them at the rate of 3 feet per second. Choose the linear equation that describes the distance from the bumper cars as a function of time elapsed, x , in seconds. Explain your reasoning.

a) $y = 30x + 3$ b) $y = 30 + 3x$ c) $y = 3 - 30x$ d) $y = 30 - 3x$

For each answer not chosen, describe a situation that corresponds to the equation.

5. Notice that the equation can always be written in the following form: Distance after x seconds equals initial distance \pm rate of motion times x . When is the sign positive, and when is it negative? Explain why. What indicates rate of motion in the graph? In the table?

6. Determine the equations for the graphs given in problem 4 on sheet 2. Be sure that the table entries that you gave on sheet 3, question 4, satisfy the appropriate equations.

7. Challenge: Find the equations for the graphs given in question 6 on sheet 2. These graphs consist of more than one line segment, so give the interval for each equation. (For example, a possible answer might be $2x + 1$ for $0 \leq x \leq 3$. Be sure that the table entries that you gave on sheet 3, question 5, satisfy the equations for the given intervals.